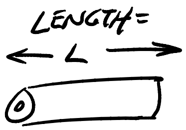
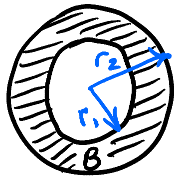
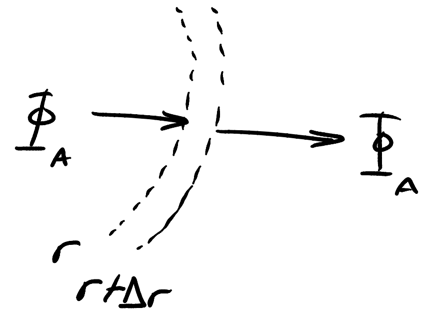
i)





REPRESENTATIVE "SLICE"



NOTE: I SUSPECT FLUX WILL BE IN OPPOSITE DIRECTION AND THUS BE NEGATIVE.

ii) SHELL MOLE BALANCE

$$\cancel{ACC}^{0} = IN - OUT + \cancel{GEN}^{0}$$

$$0 = S\Phi_A\big|_r - S\Phi_A\big|_{r+\Delta r}$$

S = CROSS SECTIONAL AREA

$$S = 2\pi r L$$

SO $$0 = 2\pi r L\,\Phi_A\big|_r - 2\pi r L\,\Phi_A\big|_{r+\Delta r}$$

DIVIDE BY $-2\pi\Delta r L$

$$0 = \frac{r\Phi_A\big|_{r+\Delta r} - r\Phi_A\big|_r}{\Delta r}$$

TAKE LIMIT AS $\Delta r \to 0$

$$\boxed{\frac{d}{dr}\left(\Phi_A r\right) = 0}$$

RESULT OF MATERIAL BALANCE

iii) RATE EXPRESSION

FICK'S LAW: $\vec{\Phi}_A = -cD_{AB}\vec{\nabla}x_A + x_A \sum_{ALL} \vec{\Phi}_j$

IN THIS PROBLEM:

1) CYLINDRICAL COORDINATES $\vec{\nabla} = \varepsilon_r \frac{\partial}{\partial r} + \cancel{\varepsilon_\theta \frac{1}{r}\frac{\partial}{\partial \theta}}^{\,0} + \cancel{\varepsilon_z \frac{\partial}{\partial z}}^{\,0}$

2) $\vec{\Phi}_B = 0$ (B IS STAGNANT)

3) $x_A \sim 0$ (DILUTE A IN SHELL)

SO $\boxed{\Phi_A = -cD_{AB}\frac{dx_A}{dr}}$

COMBINING EQUATIONS,

$$-\frac{d}{dr}\left[rcD_{AB}\frac{dx_A}{dr}\right] = 0$$

BOUNDARY CONDITIONS

@ $r = r_1$ $x_A = x_{A1}$

@ $r = r_2$ $x_A = x_{A2}$

INTEGRATE ONCE:

$$rcD_{AB}\frac{dx_A}{dr} = C_1$$

NOTE: $\Phi_A = \frac{-C_1}{r}$

INTEGRATE TWICE:

$$cD_{AB}\int dx_A = \int \frac{C_1}{r}\,dr$$

$$\underline{cD_{AB}x_A = C_1 \ln r + C_2}$$

APPLY BOUNDARY CONDITIONS:

@ $r = r_1 \quad x_A = x_{A1} \qquad cD_{AB}\, x_{A1} = C_1 \ln r_1 + C_2$

@ $r = r_2 \quad x_A = x_{A2} \qquad cD_{AB}\, x_{A2} = C_1 \ln r_2 + C_2$

SUBTRACT EQUATIONS: $\quad cD_{AB}(x_{A1} - x_{A2}) = C_1 \ln(r_1/r_2)$

$$C_1 = \frac{cD_{AB}(x_{A1} - x_{A2})}{\ln(r_1/r_2)}$$

$$C_2 = cD_{AB}\, x_{A1} - \frac{cD_{AB}(x_{A1} - x_{A2}) \ln r_1}{\ln(r_1/r_2)}$$

⋮

AFTER INSERTING C_1 & C_2 INTO EQUATION AND SIMPLIFYING

$$\boxed{x_A - x_{A1} = \frac{x_{A1} - x_{A2}}{\ln(r_1/r_2)} \ln(r/r_1)}$$

CONCENTRATION PROFILE

$$\Phi_A = -\frac{C_1}{r}$$

$$\boxed{\Phi_A = \frac{-cD_{AB}(x_{A1} - x_{A2})}{r \ln(r_1/r_2)}}$$

FLUX

iv) TOTAL MOLAR FLOW @ $r = r_2$

$$\dot{n}_A = S \cdot \Phi_A$$

$$\dot{n}_A = (2\pi r L)\left(\frac{-cD_{AB}(x_{A1} - x_{A2})}{r \ln(r_1/r_2)}\right)$$

$$\boxed{\dot{n}_A = \frac{-2\pi L c D_{AB}(x_{A1} - x_{A2})}{\ln(r_1/r_2)}}$$

SAME AT ANY VALUE OF r

v) MINIMUM LENGTH OF TUBING CORRESPONDS WITH MAXIMUM POSSIBLE FLUX OR MOLAR FLOW. THIS OCCURS WHEN $c_{A1} = 0$.

$$\text{SO} \quad \dot{n}_A(\text{max}) = \frac{2\pi L D_{AB} c_{A2}}{\ln(r_1/r_2)}$$

$$\dot{n}_A = \frac{24\,\text{mg}}{\text{h}} \times \frac{\text{h}}{3600\,\text{s}} \times \frac{\text{g}}{1000\,\text{mg}} \times \frac{\text{mol}}{46\,\text{g}} = 1.47 \times 10^{-7}\ \text{mol/s}$$

$$D_{AB} = 4.2 \times 10^{-5}\ \text{cm}^2/\text{s}$$

$$c_{A2} = \frac{21\,\text{mg}}{\text{L}} \times \frac{\text{g}}{1000\,\text{mg}} \times \frac{\text{mol}}{46\,\text{g}} \times \frac{\text{L}}{1000\,\text{cm}^3} = 4.565 \times 10^{-7}\ \text{mol/cm}^3$$

$d_2 = 0.83\,\text{cm} \qquad r_2 = 0.415\,\text{cm}$

$d_1 = 0.61\,\text{cm} \qquad r_1 = 0.305\,\text{cm}$

SO

$$L = \frac{n_A(\text{MAX}) \ln(r_2/r_1)}{2\pi D_{AB}\, c_{A2}} = \frac{(1.47\times10^{-7}\ \text{mol/s}) \ln\left(\frac{0.415}{0.305}\right)}{2\pi\,(4.2\times10^{-5}\ \text{cm}^2/\text{s})(4.565\times10^{-7}\ \text{mol/cm}^3)}$$

$$= \underline{371\ \text{cm}}$$

CONC. PROFILE IS

$$c_A - c_{A1} = \frac{c_{A1} - c_{A2}}{\ln(r_1/r_2)} \ln(r/r_1)$$

OR

$$c_A = \frac{-21}{\ln(3.05/4.15)} \ln(r/3.05) = 68.19 \ln(r/3.05)$$

WHERE r IS IN [mm]
AND c_A IS IN [ppm]

r (mm)		c_A (ppm)
3.05		0
3.15		2.2
3.25		4.3
3.35		6.4
3.45		8.4
3.55		10.4
3.60	MIDPOINT	11.3
⋮		⋮